Propagation of the Quasi-TEM Mode in Ferrite-Filled Coaxial Line

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Summary—**The Suhl and Walker approximation for the propagation constant of the quasi-TEM mode in ferrite-filled parallel plane waveguide has been applied to the ferrite-filled coaxial line. The approximation is compared to exact solutions for a coaxial line filled with a lossless ferrite with close agreement.**

The propagation constant of the quasi-TEM mode is determined by measuring the complex reflection coefficient of a plane ferrite-air interface. The α and β are compared to the Suhl and Walker approximation with losses, and qualitative agreement is found.

In order to relate the measured values to the propagation constants, the boundary value problem of the reflection from a plane ferrite-air interface is investigated. Expressions are derived which relate the real and imaginary parts of the propagation constant in the ferrite to an approximation to the complex reflection coefficient of the TEM mode in the empty line.

Introduction

A NUMBER OF AUTHORS have described devices constructed by completely filling a coaxial line with a ferrite and applying an axial magnetic field. These devices are proposed for use as filters [1], switches [2]–[4], and phase shifters [5], [6] at S and L bands. Since no theoretical results were available for the ferrite-filled coaxial line,[1] these authors interpreted the behavior of their devices by means of the Suhl and Walker approximation [7] for the closely spaced, ferrite-filled, parallel plane waveguide.

The Suhl and Walker approximation was derived by considering a parallel plane waveguide filled with a lossless ferrite biased by a magnetic field in the direction of propagation. When the spacing between the conducting planes is small, the propagation constant for the quasi-TEM mode is given by a simple expression. The argument for applying the results of an analysis of the parallel plane waveguide to the coaxial waveguide is that the coaxial line can be "unrolled" [4] to give a parallel plane system.

Kales [8] and Epstein [9] have presented theoretical techniques for solving the boundary value problem. Kales did not consider the ferrite-filled coaxial line in detail, but Epstein developed a determinantal equation for the propagation constant. We have solved the determinantal equation for the quasi-TEM mode by numerical methods.

The deviation of the approximation from the exact solutions is presented as a function of magnetic field for a given spacing between the conductors and as a function of spacing for a given magnetic field.

The propagation constant in a ferrite filled coaxial line is measured for several frequencies as a function of magnetic field. The technique used to measure the propagation constant is to observe the reflection of the TEM mode from a plane ferrite-air interface. The results of the measurements are then compared to the Suhl and Walker approximation when loss terms are included.

The fields of the quasi-TEM mode at a ferrite-air interface are expanded in terms of the empty coaxial line modes. The relation between the propagation constant in the ferrite and an approximation to the TEM mode reflection coefficient at the interface is found.

Theoretical Solutions for the Propagation Constant

The general field equations for any mode in a ferrite-filled coaxial line are given in Appendix I. Setting $n=0$ in the general field equations yields the expressions for the quasi-TEM mode and all higher symmetrical modes. The propagation constant of the quasi-TEM mode is found by solving the determinantal equation for spacings between the inner and outer conductors so small that all higher modes are cut off.

The tangential electric fields for the quasi-TEM mode and all higher symmetrical modes are

$$E_z = \sum_{i=1}^{2} S_i^2\{C_iY_0(S_ir) + \overline{C}_i\overline{Y}_0(S_ir)\}$$

$$E_\phi = \sum_{i=1}^{2} \frac{j\mu(S_i^2 - a)}{\gamma k} S_i\{C_iY_0'(S_ir) + \overline{C}_i\overline{Y}_0'(S_ir)\}. \quad (1)$$

If the inner radius is R_1 and the outer radius is R_2, the boundary conditions are that the tangential electric field be zero at $r=R_1$ and $r=R_2$. Applying the boundary conditions yields a set of four linear, homogeneous equations in the coefficients C_i and $\overline{C}_i$. In order to find

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[1] Sandler has developed approximate solutions for the quasi-TE_{11} mode. S. E. Sandler, "An approximate solution to some ferrite-filled waveguide problems with longitudinal magnetization," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-9, pp. 162–168; March, 1961.

nontrivial solutions, the determinant of the coefficients of the C_i's and $\overline{C}_i$'s must be equal to zero. The determinantal equation is therefore

$$\begin{vmatrix} S_1^2 Y_0(S_1R_1) & S_1^2 \overline{Y}_0(S_1R_1) & S_2^2 Y_0(S_2R_1) & S_2^2 \overline{Y}_0(S_2R_1) \\ S_1^2 Y_0(S_1R_2) & S_1^2 \overline{Y}_0(S_1R_2) & S_2^2 Y_0(S_2R_2) & S_2^2 \overline{Y}_0(S_2R_2) \\ (S_1^2 - a)S_1 Y_0'(S_1R_1) & (S_1^2 - a)S_1 \overline{Y}_0'(S_1R_1) & (S_2^2 - a)S_2 Y_0'(S_2R_1) & (S_2^2 - a)S_2 \overline{Y}_0'(S_2R_1) \\ (S_1^2 - a)S_1 Y_0'(S_1R_2) & (S_1^2 - a)S_1 \overline{Y}_0'(S_1R_2) & (S_2^2 - a)S_2 Y_0'(S_2R_2) & (S_2^2 - a)S_2 \overline{Y}_0'(S_2R_2) \end{vmatrix} = 0. \quad (2)$$

Eq. (2) is a transcendental equation in the propagation constant since the separation constants, S_1 and S_2, are functions of the material parameters and the propagation constant. The zeros of the determinant will occur for the values of the propagation constants of the symmetrical modes.

It should be noted that there are two false zeros of (2). If either $S_1 = 0$ or $S_2 = 0$, this equation is identically zero. However, except for discrete values of R_1 and R_2, the C_i's and $\overline{C}_i$'s must be zero to satisfy the boundary conditions. These false zeros correspond to Kales' trivial solutions.

For the lossless case, the separation constants are either pure real or pure imaginary. In the former, the unmodified Bessel functions are used and in the latter, the modified Bessel functions. When losses are included, the separation constants become complex, and thus each $Y_n(x)$ and $\overline{Y}_n(x)$ also becomes complex. No attempt was made to consider this case exactly.

Numerical solution of (2) was carried out on an IBM 709 digital computer. The material parameters for a commercially available ferrite, Trans-Tech TT-414, were used ($\epsilon_r = 11.5$, $4\pi M_s = 680$ Gauss), the inner and outer radii were chosen for 50 ohm coaxial line ($R_1 = 0.125$ inch and $R_2/R_1 = 2.25$), and the frequency was 1.5 Gc. The computations were made by holding the frequency, material parameters, geometry, and magnetic field constant and computing values of (2) for small steps of propagation constant. The magnetic field and propagation constant were normalized to the resonant field and $j\omega\sqrt{\mu_0\epsilon}$ respectively. The solutions were found by noting the interval of propagation constant in which the determinant changed sign. Then the Suhl and Walker approximation, given by

$$\gamma_n = \left[\frac{\mu^2 - k^2}{\mu}\right]^{1/2} \quad (3)$$

and shown in Fig. 1, was compared to the exact solutions. Exact solutions were calculated as a function of magnetic field for constant spacing between conductors and as a function of spacing for constant magnetic field.

When $R_1 = 0.125$ inch and $R_2/R_1 = 2.25$, the Suhl and Walker approximation was found to deviate from the exact solutions by 3.7 per cent for $H_n = 0.75$ and by less than 0.03 per cent for $H_n = 3.0$.

When $R_1 = 0.125$ inch and $H_n = 1.2$, the Suhl and Walker approximation was found to deviate from the exact solutions by less than 0.5 per cent for $R_2/R_1 = 2.25$ and by 6 per cent for $R_2/R_1 = 6.5$. The latter spacing is that for which higher order symmetrical modes begin to appear. For the same inner radius and for $H_n = 2.0$, the Suhl and Walker approximation deviated from the exact solutions by less than 0.2 per cent for $R_2/R_1 = 2.25$ and by less than 0.75 per cent for $R_2/R_1 = 7.5$, the spacing at which higher symmetrical modes appear for this magnetic field.

The results of the numerical analysis show that the Suhl and Walker equation is a close approximation to the propagation constant of the quasi-TEM mode for the lossless case. The approximation is especially valid for magnetic fields much larger than the resonant field, and for close spacing between inner and outer conductors.

However, losses are always present to some extent, and, in the vicinity of the resonant field, the loss terms may become large. Therefore, as a first approximation to the lossy case, phenomenological loss terms will be placed in the Suhl and Walker equation.

When losses are present, the elements of the μ tensor become complex.

$$\mu = \mu' - j\mu''$$
$$k = k' - jk''. \quad (4)$$

The real and imaginary parts can be calculated by means of the expressions given by Lax and Button (see page 154 [10]).

When the losses as given in (4) are substituted into (3), the phase constant behaves as in Fig. 2. The result for a linewidth of 100 oersteds (the nominal linewidth of TT-414 ferrite measured at 3 Gc) is compared to the lossless approximation. The effect of placing loss terms in (3) eliminates the low-field cutoff region. When larger linewidths are used, the β-H curve is smoothed out.

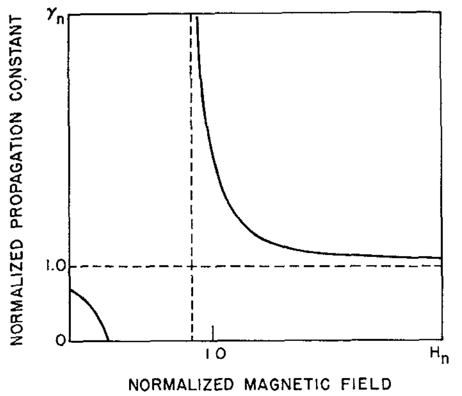

Fig. 1—The Suhl and Walker approximation for the lossless case.

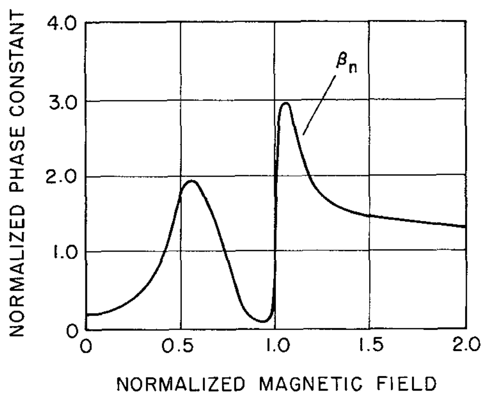

Fig. 2—The Suhl and Walker approximation with losses. Linewidth = 100 oersteds.

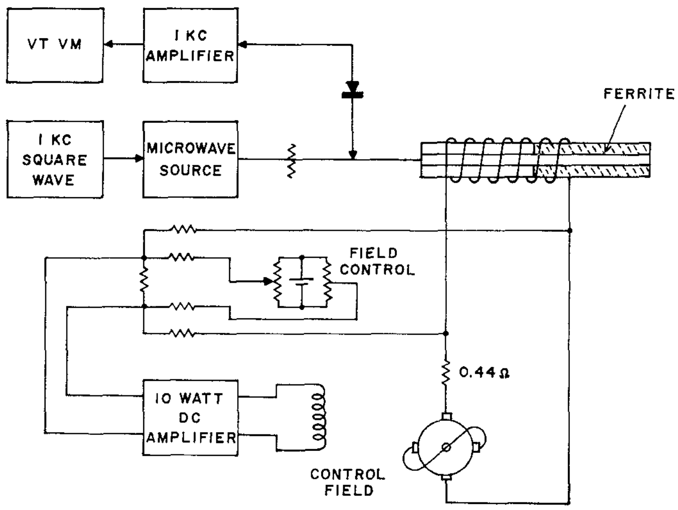

Fig. 3—Experimental apparatus.

Experimental Measurement of the Propagation Constant

The experimental investigation was conducted to find the range of magnetic fields in which the Suhl and Walker approximation describes the behavior of a coaxial line filled with a lossy ferrite. The propagation constant was measured in a ferrite-filled section and compared to the approximation.

Two techniques that were tried were: 1) measuring the standing wave in a section of ferrite-filled line, and 2) observing the resonances of a ferrite-filled coaxial transmission cavity. These techniques failed when large losses were present.

The chosen experimental technique, used to measure the propagation constant, was to observe the reflection of the TEM mode from a plane ferrite-air interface. A ferrite sample with plane endfaces was placed in the coaxial line with a slotted line two or more wavelengths from the sample. The VSWR and position of the minimum were measured in the slotted line as the magnetic field was varied. In Appendix II the relations between the approximate complex TEM mode reflection coefficient of the interface and the propagation constant of the quasi-TEM mode in the ferrite are derived. The magnitude and angle of the reflection coefficient were reduced by means of the results of Appendix II, (21) and (22), to yield the real and imaginary parts of the quasi-TEM mode propagation constant in the ferrite.

The results from this method were compared to the wavelengths measured using a ferrite-filled coaxial transmission cavity, and agreement was found.

Since multiple reflections would introduce errors, it was necessary to terminate the ferrite-filled section. The method of termination was to place only the first endface of the ferrite sample in the magnetic field. The sample, which was 19 cm long, was allowed to protrude from the end of the biasing solenoid. Therefore, the terminal end of the ferrite was placed in low-field, high-loss operation. The absence of multiple reflections was checked by changing the position of a sliding short in the line past the terminal end of the ferrite. No significant variation of the VSWR in the line in front of the sample was noted.

The experimental investigation was carried out for frequencies in the range of 0.6 to 2.0 Gc. The coaxial line had an inner diameter of $\frac{1}{4}$ inch and an outer diameter of 9/16 inch and was filled with Trans-Tech type TT-414 ferrite. The microwave source was 1000 cycle square wave modulated, and the receiving system included a tuned, 1000 cycle narrow-band amplifier. Fig. 3 shows the experimental arrangement.

The current which produced the biasing field was controlled by a GE 1.5 kw amplidyne with appropriate feedback. This arrangement made it possible to vary the current in the load from 0 to 10 amperes by changing the current in the control windings of the amplidyne from 0 to 15 ma.

The axial magnetic field was varied from 0 to 1000 oersteds. This biasing field was produced by a solenoid 18 inches long wound on a water jacket with an inner diameter of 2 inches. A current of about 10 amperes was necessary to produce a field of 1000 oersteds. Sustained

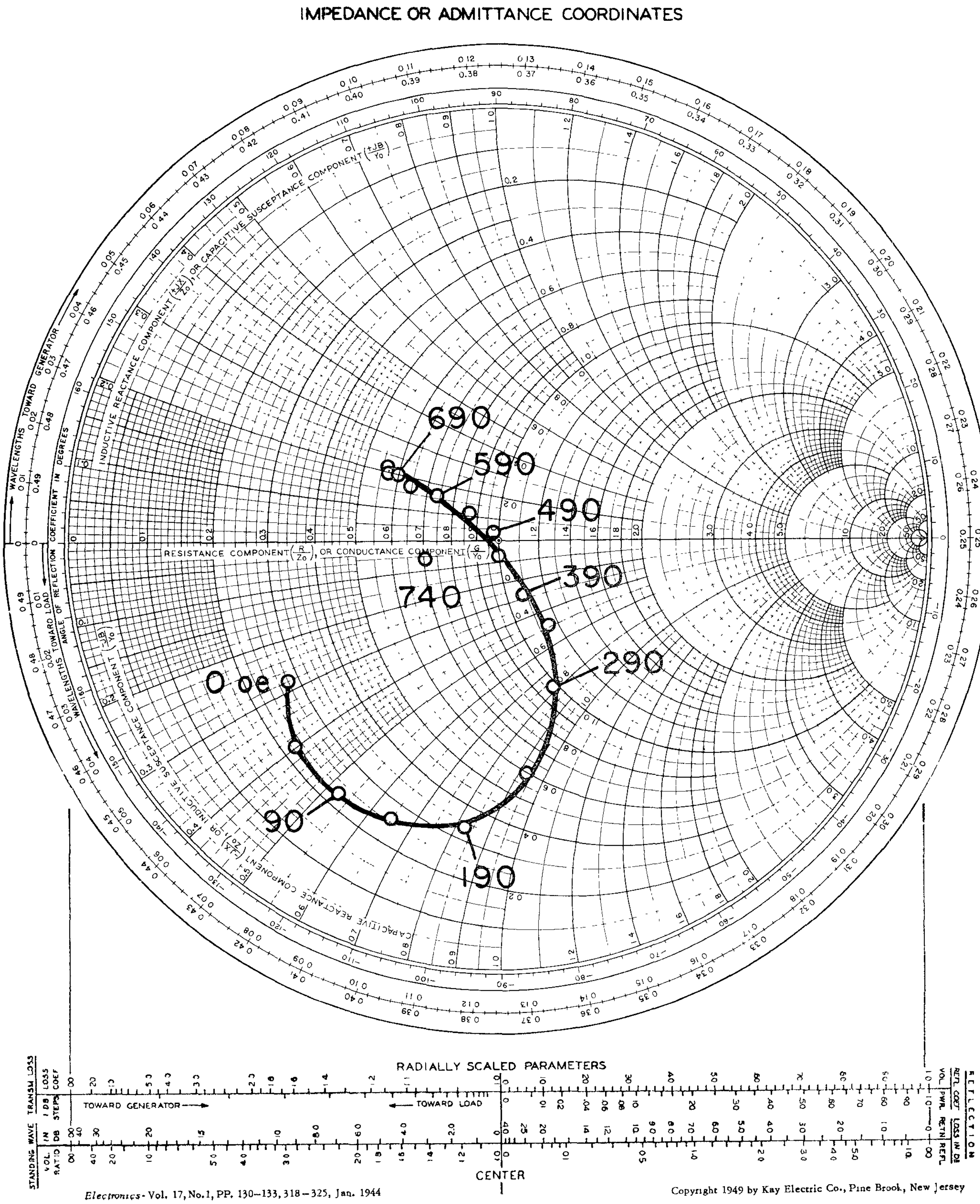

Fig. 4—Reflection of the TEM mode from a ferrite-air interface.

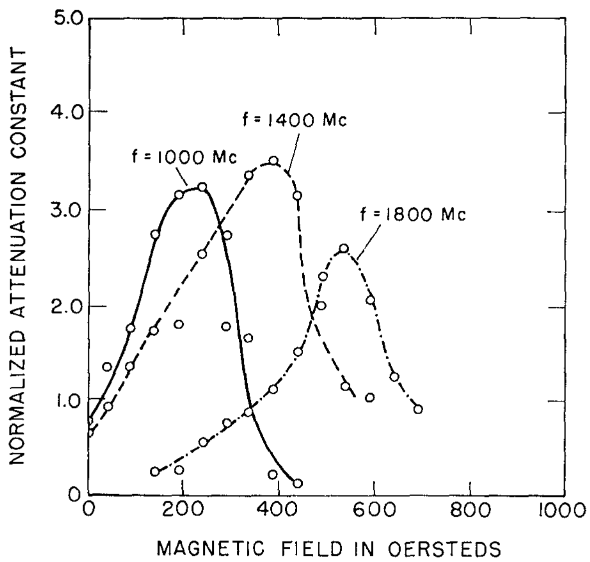

Fig. 5—Comparison of α measured for 1.0, 1.4, and 1.8 Gc.

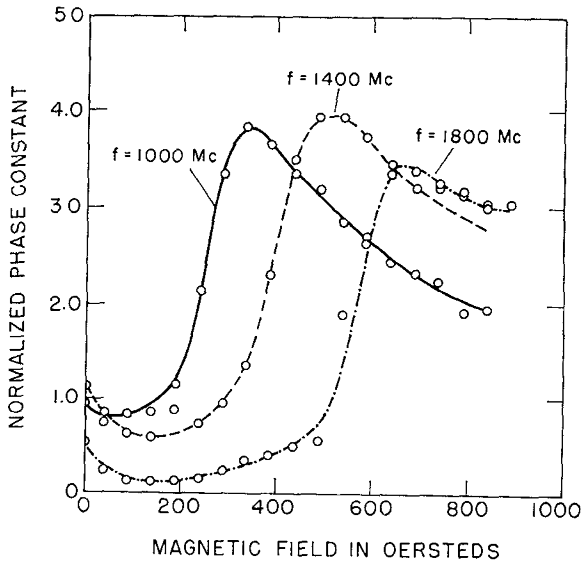

Fig. 6—Comparison of β measured for 1.0, 1.4, and 1.8 Gc.

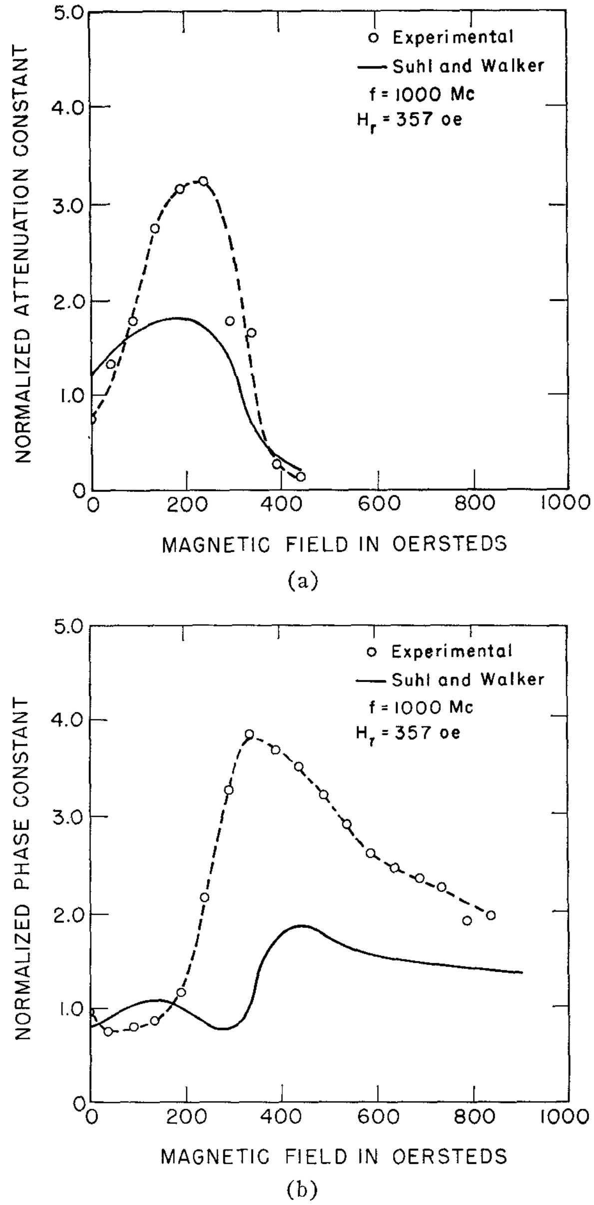

Fig. 7—(a) Comparison of measured α to Suhl and Walker approximation. (b) Comparison of measured β to Suhl and Walker approximation.

operation at high currents was possible only when the solenoid was packed in ice.

In order to minimize the effects of incomplete saturation, the ferrite was saturated by first raising the magnetic field to 1000 oersteds and then reducing it to zero. The field was then increased in steps of 50 oersteds and the complex reflection coefficient was measured. The results for a typical run are shown on a Smith chart in Fig. 4.

As the magnetic field increases from zero to well above resonance, the VSWR first increases, then decreases, to the neighborhood of 1.0, and finally increases again. The phase of the complex reflection coefficient changes in a counterclockwise direction on the Smith chart as the magnetic field increases. The initial and final phases varied as a function of frequency. In most cases, the initial phase moved clockwise as the frequency was raised.

The data were reduced by means of (21) and (22) to find the real and imaginary parts of the propagation constant in the ferrite. The attenuation constant is shown in Fig. 5 and the phase constant in Fig. 6. As expected, the magnetic field for maximum loss and the magnetic field for most rapid change of the phase constant were found to increase with increasing frequency.

The results for 1.0 Gc are compared to the lossy Suhl and Walker approximation in Figs. 7(a) and 7(b). TT-414 ferrite has a linewidth of 100 oersteds at S band. However, at 1.0 Gc, in a coaxial line, the ferrite behaves as if the linewidth were about 300 oersteds.

The experimental α, shown in Fig. 7(a), is larger than the approximate theoretical α. Both curves extend over the same range of magnetic fields, though the experimental curve has a narrower maximum at a slightly higher magnetic field than the approximate curve.

The phase constant is shown in Fig. 7(b). In the low field region, below 200 oersteds, the approximate and experimental values are in substantial agreement. The experimental curve shows a rapid increase in β from 200 to 350 oersteds, whereas the approximate theory predicts a rapid increase between 300 and 400 oesteds. The experimental data above 400 oersteds decreases with a slope that approaches the theoretical slope.

Conclusions

The theoretical solutions for the propagation constant of the quasi-TEM mode for the lossless case show that the Suhl and Walker equation is a close approximation to this case. However, the propagation constant measured experimentally shows qualitative agreement with the lossless approximation only for magnetic fields several times the resonant field. When loss terms are placed in the approximation, the result agrees qualitatively with the experimental results for all magnetic fields as in Fig. 7, though quantitative disagreement is noted.

Several ferrite-filled coaxial line devices were referred to in the introduction of this paper. The phase shifters, being transmission devices, are operated in ranges of magnetic field for which α is small. For these magnetic fields, the lossless approximation holds, and can be expected to produce good results. The switches are operated between ranges of magnetic field for which α is large, and fields for which α is small. Since the range of fields in which the lossless approximation predicts cutoff is very nearly the same as the range in which the lossy approximation predits large α and in which the measured α is large, the lossless approximation can be used successfully to design switches.

Appendix I

General Field Equations for a Ferrite-Filled Coaxial Line

The theoretical solutions for ferrite-filled cylindrical waveguide with axial magnetization are well known [8]–[11]. The special case of the completely filled coaxial line is presented explicitly by Epstein [9] and is implied by Kales [8]. The equations for the fields in a ferrite-filled coaxial line with axial magnetization are given below in Kales' notation, and for convenience, the separation constants S_1 and S_2 are defined as the square roots of Kales' separation constants. The functions $Y_n(x)$ and $\overline{Y}_n(x)$ are Bessel functions of the first and second kinds respectively.

$$E_z = \sum_{i=1}^{2} S_i^2[C_iY_n(S_ir) + \overline{C}_i\overline{Y}_n(S_ir)]e^{\pm jn\phi}$$

$$H_z = \sum_{i=1}^{2} \frac{S_i^2 - a}{b} S_i^2[C_iY_n(S_ir) + \overline{C}_i\overline{Y}_n(S_ir)]e^{\pm jn\phi}$$

$$E_r = \sum_{i=1}^{2}\left\{\frac{S_i^2 - a}{b}\frac{\mu n}{\gamma k}[C_iY_n(S_ir) + \overline{C}_i\overline{Y}_n(S_ir)] - \gamma S_i[C_iY_n'(S_ir) + \overline{C}_i\overline{Y}_n'(S_ir)]\right\}e^{\pm jn\phi}$$

$$E_\phi = \sum_{i=1}^{2}\left\{\frac{-j\gamma n}{r}[C_iY_n(S_ir) + \overline{C}_i\overline{Y}_n(S_ir)] + \frac{j\mu(S_i^2 - a)}{\gamma k}S_i[C_iY_n'(S_ir) + \overline{C}_i\overline{Y}_n'(S_ir)]\right\}e^{\pm jn\phi}$$

$$H_r = \sum_{i=1}^{2}\left\{-n\omega\epsilon[C_iY_n(S_ir) + \overline{C}_i\overline{Y}_n(S_ir)] + (\beta^2 + \gamma^2 - S_i^2)\frac{S_i}{\omega k}[C_iY_n'(S_ir) + \overline{C}_i\overline{Y}_n'(S_ir)]e\right\}e^{\pm jn\phi}$$

$$H_\phi = \sum_{i=1}^{2}\left\{(\beta^2 + \gamma^2 - S_i^2)\frac{jn}{\omega kr}[C_iY_n(S_ir) + \overline{C}_i\overline{Y}_n(S_ir)] - j\omega\epsilon S_i[C_iY_n'(S_ir) + \overline{C}_i\overline{Y}_n'(S_ir)]\right\}e^{\pm jn\phi}$$

$$S_i^2 = \frac{a + c \pm \sqrt{(a-c)^2 + 4bd}}{2}$$

$$a = \beta^2 + \gamma - \beta'^2\frac{k}{\mu}$$

$$b = \omega\mu_z\gamma\frac{k}{\mu}$$

$$c = (\beta^2 + \gamma^2)\frac{\mu_z}{\mu}$$

$$d = -\omega\epsilon\gamma\frac{k}{\mu}$$

where $\beta^2 = +\omega^2\mu\epsilon$, $\beta'^2 = +\omega^2k\epsilon$, and μ, k, and μ_z are elements of μ tensor.

Appendix II

An Approximation to the Reflection of the TEM Mode from a Ferrite-Air Interface

A plane ferrite-air interface, S, is located at $z=0$ in a coaxial line as shown in Fig. 8. The ferrite is saturated by a z-directed magnetic field, and a TEM wave is incident upon the interface in the empty coaxial line. The ferrite-filled section is assumed to be terminated with no reflections.

At the surface S, the tangential fields in the ferrite will be expressed in terms of the tangential components of the empty coaxial line modes. The product of the fields with the TEM mode conjugate will be formed, and the fields will be integrated across the surface. The coefficients of the TEM mode, A_0 and B_0, will be approximated by neglecting all modes in the ferrite except the quasi-TEM mode. The approximate reflection coefficient of the TEM mode will be expressed in terms of A_0 and B_0. The relationship between the approximate

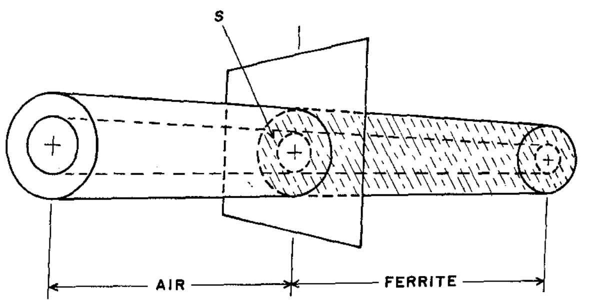

Fig. 8—A plane ferrite-air interface in a coaxial line.

form of the complex reflection coefficient and the propagation constant of the quasi-TEM mode in the ferrite will be established.

At the surface S, the tangential componets of $\overline{E}$ and $\overline{H}$ must be continuous. Since the incident wave is symmetrical, and since the interface introduces no asymmetries, only symmetrical modes will be excited in the ferrite and reflected from the interface. The superscript f will refer to the fields in the ferrite, and the subscript t will denote the tangential components.

$$\sum_{n=1}^{\infty} T_n(\overline{E}_n^f)_t = (A_0+B_0)(\overline{E}_{\text{TEM}})_t + \sum_{n=1}^{\infty} B_n(\overline{E}_n)_t \quad (5)$$

$$\sum_{n=1}^{\infty} T_n(\overline{H}_n^f)_t = (A_0-B_0)(\overline{H}_{\text{TEM}})_t - \sum_{n=1}^{\infty} B_n(\overline{H}_n)_t. \quad (6)$$

We form the scalar product of both sides of (5) with $(\overline{E}_{\text{TEM}})_t^*$ and integrate across S.

$$\sum_{n=1}^{\infty} T_n \int_0^{2\pi}\int_{R_1}^{R_2} (\overline{E}_{\text{TEM}})_t^* \cdot (\overline{E}_n^f)_t r dr d\phi = (A_0+B_0)\int_0^{2\pi}\int_{R_1}^{R_2} |(E_{\text{TEM}})_t|^2 r dr d\phi. \quad (7)$$

The tangential component of the TEM electric field is the r component.

$$(\overline{E}_{\text{TEM}})_t = \overline{E}_r = \frac{\bar{a}_r E_0}{2\pi r} e^{j\omega t - \gamma z}. \quad (8)$$

Eq. (8) is substituted into (7). We integrate the right-hand side, and perform the ϕ integration on the left-hand side.

$$\sum_{n=1}^{\infty} E_0 T_n \int_{R_1}^{R_2} (E_n^f)_r dr = \frac{(A_0+B_0)}{2\pi} E_0^2 \ln \frac{R_2}{R_1}. \quad (9)$$

Since the tangential component of the TEM magnetic field is the ϕ component, which can be expressed as $H_\phi = E_r/\sqrt{\mu_0/\epsilon_0}$, (6) leads to a similar result.

$$\sum_{n=1}^{\infty} T_n \frac{E_0}{\sqrt{\frac{\mu_0}{\epsilon_0}}} \int_{R_1}^{R_2} (H_n^f)_\phi dr = \frac{(A_0-B_0)}{2\pi \frac{\mu_0}{\epsilon_0}} E_0^2 \ln \frac{R_2}{R_1}. \quad (10)$$

To find the symmetrical mode field expressions we set n equal to zero in the equations in Appendix I. Therefore $(E^f)_r$ and $(H^f)_\phi$ are

$$(E^f)_r = -\gamma_f \sum_{i=1}^{2} S_i(C_i Y_0'(S_i r) - \overline{C}_i \overline{Y}_0'(S_i r))$$

$$(H^f)_\phi = -j\omega\epsilon_f \sum_{i=1}^{2} S_i(C_i Y_0'(S_i r) + \overline{C}_i \overline{Y}_0'(S_i r)). \quad (11)$$

Eqs. (11) show that $(H^f)_\phi = [j\omega\epsilon_f/\gamma_f](E^f)_r$. Therefore, (9) and (10) can be written in terms of the integral,

$$I_n = \int_{R_1}^{R_2} (E_n^f)_r dr.$$

$$\sum_{n=1}^{\infty} T_n E_0 I_n = \frac{(A_0+B_0)}{2\pi} E_0^2 \ln \frac{R_2}{R_1}$$

$$\sum_{n=1}^{\infty} T_n \frac{j\omega\epsilon_f}{\gamma_f} E_0 I_n = \frac{(A_0-B_0)}{2\pi \frac{\mu_0}{\epsilon_0}} E_0^2 \ln \frac{R_2}{R_1}. \quad (12)$$

Since this problem leads to an infinite set of simultaneous linear equations [9], we shall make an approximation by neglecting all modes in the ferrite except the quasi-TEM mode. The results obtained by applying the approximate form of the reflection coefficient to experimental data are consistent with independent measurements of the wavelength in a ferrite-filled transmission cavity.

Solving for A_0 yields

$$A_0 \approx \frac{T_1 I_1}{2}\left\{\frac{1}{2\pi E_0 \ln \frac{R_2}{R_1}} + \frac{j\omega\epsilon_f\sqrt{\frac{\mu_0}{\epsilon_0}}}{\gamma_f 2\pi E_0 \ln \frac{R_2}{R_1}}\right\} \quad (13)$$

and solving for B_0 yields

$$B_0 \approx \frac{T_1 I_1}{2}\left\{\frac{1}{2\pi E_0 \ln \frac{R_2}{R_1}} - \frac{j\omega\epsilon_f\sqrt{\frac{\mu_0}{\epsilon_0}}}{\gamma_f 2\pi E_0 \ln \frac{R_2}{R_1}}\right\}. \quad (14)$$

The approximate reflection coefficient R_0 is given by the ratio of B_0 to A_0.

$$R_0 = \frac{1 - \frac{j\omega\epsilon_f}{\gamma_f}\sqrt{\frac{\mu_0}{\epsilon_0}}}{1 + \frac{j\omega\epsilon_f}{\gamma_f}\sqrt{\frac{\mu_0}{\epsilon_0}}}. \quad (15)$$

Multiplying the numerator and denominator by γ_f and dividing by $j\omega\sqrt{\mu_0\epsilon_f}$, (15) becomes

$$R_0 = \frac{\gamma_{fn} - \sqrt{\epsilon_{f\,\text{rel.}}}}{\gamma_{fn} + \sqrt{\epsilon_{f\,\text{rel.}}}}. \quad (16)$$

Eq. (16) gives the approximate reflection coefficient of the TEM mode from a plane ferrite-air interface for the quasi-TEM mode in the ferrite. γ_{fn} is the propagation constant of the mode in the ferrite normalized to the propagation constant of the TEM mode in a coaxial line filled with a dielectric with the same dielectric constant as the ferrite. No assumptions have been made reregarding the losses in the development of (16).

We will now find the relationship between the real and imaginary parts of the propagation constant and the complex reflection coefficient. When losses are present the propagation constant in the ferrite is given by $\gamma=\alpha+j\beta$. Dividing both sides by $j\omega\sqrt{\mu_0\epsilon_f}$ and normalizing α and β with respect to $\omega\sqrt{\mu_0\epsilon_f}$ leads to

$$\gamma_n = -j\alpha_n + \beta_n. \tag{17}$$

We substitute (17) into (16).

$$R_0 = \frac{\beta_n - \sqrt{\epsilon_r} + j\alpha_n}{\beta_n + \sqrt{\epsilon_r} - j\alpha_n}\cdot \tag{18}$$

When (18) is rationalized, the real and imaginary parts of the TEM mode reflection coefficient are given by

$$R_r = \frac{\beta_n^2 - \epsilon_r + \alpha_n^2}{(\beta_n + \sqrt{\epsilon_r})^2 + \alpha_n^2}, \tag{19}$$

$$R_i = \frac{-2\alpha_n\sqrt{\epsilon_r}}{(\beta_n + \sqrt{\epsilon_r})^2 + \alpha_n^2}\cdot \tag{20}$$

Eq. (19) can be solved for α.

$$\alpha_n = \pm\left\{\frac{\beta_n^2 - \epsilon_r - (\beta_n + \sqrt{\epsilon_r})^2 R_r}{R_r - 1}\right\}^{1/2}. \tag{21}$$

Substituting (21) into (20) and solving for β_n leads to

$$\beta_n = \frac{1 - R_r^2 - R_i^2}{R_i^2 + (R_r - 1)^2}\,\epsilon_r^{1/2}. \tag{22}$$

Eqs. (21) and (22) can be used to find the propagation constant of the quasi-TEM mode in a ferrite-filled coaxial line by measuring the complex reflection coefficient of the TEM mode from a plane ferrite-air interface.

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